Enrollment No: _____ Exam Seat No:_____

C.U.SHAH UNIVERSITY
Winter Examination-2022

Subject Name: Transform Methods

Subject Code: 4SC05TRM1   **Branch: B.Sc. (Mathematics)**

Semester: 5   **Date: 28/11/2022**   **Time: 02:30 To 05:30**   **Marks: 70**

Instructions:
  (1) Use of Programmable calculator & any other electronic instrument is prohibited.
  (2) Instructions written on main answer book are strictly to be obeyed.
  (3) Draw neat diagrams and figures (if necessary) at right places.
  (4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** (14)
a) Define: Laplace Transform (01)
b) State Dirichlet's conditions for Fourier series. (02)
c) Find: $L\left(\sin^2 t + \cos^2 t + 3\right)$ (02)
d) Find $1*1$ (02)
e) In the Fourier series expansion of $f(x) = \cos x$ in $(-\pi, \pi)$, the value of $b_n = $ _____. (01)
f) What is the value of a_0 in the Fourier series expansion of $f(x) = x^2$ in (1,3)? (02)
g) Define: Z-transform (02)
h) Prove that $Z(n) = \dfrac{z}{(z-1)^2}$. (02)

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** (14)
a) Solve the differential equation $\left(y'' + 3y' + 2y\right) = e^t$; $y(0) = 1, y'(0) = 0$ by using laplace transformation. (07)

b) Find: $L^{-1}\left(\log\left(1 + \dfrac{1}{s^2}\right)\right)$ (04)

c) Find Laplace transformation of $\cos 3t$ by using the definition of it. (03)

Q-3 **Attempt all questions** (14)
a) Obtain Fourier series of $f(x) = e^{ax}$ in $(-\pi, \pi)$. (05)

b) Find the Fourier series of $f(x) = \begin{cases} x, & 0 < x < \pi \\ 2\pi - x, & \pi < x < 2\pi \end{cases}$ and hence prove that   (05)

$\dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \ldots = \dfrac{\pi^2}{8}$.

c) Find the half range sine series of $f(x) = \pi - x$ in $(0, \pi)$.   (04)

Q-4 Attempt all questions   **(14)**

a) State and prove Convolution theorem.   (07)

b) Find: $L^{-1}\left(\dfrac{5s^2 + 3s - 16}{(s-1)(s-2)(s+3)} \right)$ (04)

c) State and prove First Shifting theorem.   (03)

Q-5 Attempt all questions   **(14)**

a) Find the Fourier series of $f(x) = x^3$ in $(0, 2)$.   (07)

b) Find the half range cosine series of $f(x) = (x-1)^2$ in $(0,1)$.   (04)

c) Find Z-transform of $n \sin n\theta$.   (03)

Q-6 Attempt all questions   **(14)**

a) Find $L^{-1}\left(\dfrac{1}{\left(s^2 + a^2\right)^2} \right)$ by using Convolution theorem.   (05)

b) Find: $L\left(t e^{-2t} \cos t \right)$ (05)

c) Evaluate: $\displaystyle\int_0^\infty \dfrac{e^{-t} \sin t}{t} \, dt$ (04)

Q-7 Attempt all questions   **(14)**

a) Find fourier integral representation of the function $f(x) = \begin{cases} -e^{kx}, & x < 0 \\ e^{-kx}, & x > 0 \end{cases}$ and   (07)

hence evaluate $\displaystyle\int_0^\infty \dfrac{\lambda \sin \lambda x}{k^2 + \lambda^2} \, d\lambda = \dfrac{\pi}{2} e^{-kx}$.

b) Find Fourier transform of $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ and hence evaluate $\displaystyle\int_0^\infty \dfrac{\sin x}{x} \, dx$.   (07)

Q-8 Attempt all questions   **(14)**

a) If $Z(u_n) = U(z)$ then $Z(u_{n-k}) = z^{-k} U(z)$, where $k > 0$ (05)

b) Find the Z-transform of $a^n \cosh n\theta$.   (05)

c) State and prove multiplication by n rule for Z-transform and also write its generalized form.   (04)